

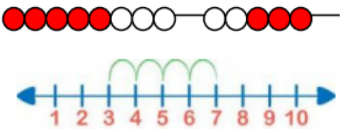
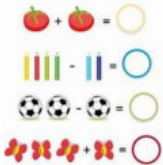


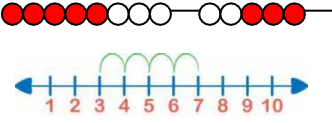






# Great Barton Primary Academy Calculation Policy

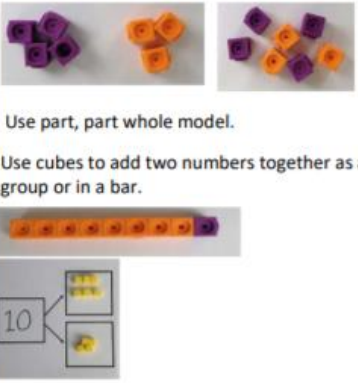
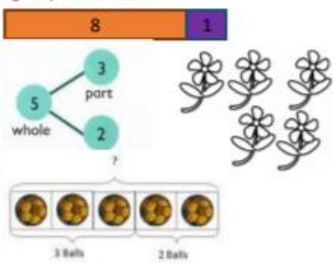


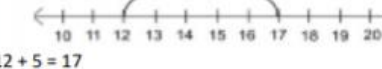
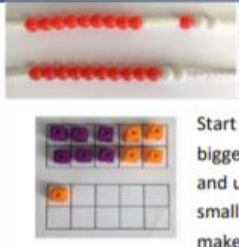
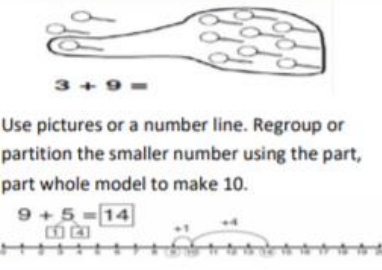

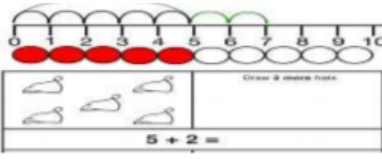


Throughout the teaching of each of the operations (addition, subtraction, multiplication and division), we use a concrete – pictorial – abstract approach to ensure pupils develop a deeper understanding of each new concept and strategy.

Early Years			
Addition	Subtraction	Multiplication	Division
<p>Children are encouraged to gain a sense of the number system through the use of counting concrete objects.</p>  <p>They combine objects in practical ways and count all.</p>  <p>They understand addition as counting on and will count on in ones and twos using objects, cubes, bead strings and number lines.</p>  <p>They use concrete and pictorial representation to record their calculations.</p> <p>They begin to use + and = They are encouraged to develop a mental picture of the number system in their heads to use for calculations.</p>  <p>Higher attaining children may be able to represent their calculations using symbols and numbers.</p>	<p>Children are encouraged to gain a sense of the number system through the use of counting concrete objects.</p>   <p>They understand subtraction as counting out.</p> <p>They begin to count back in ones and twos using objects, cubes, bead strings and number lines.</p>  <p>They use concrete and pictorial representation to record their calculations.</p> <p>They begin to use - and =</p> <p>They are encouraged to develop a mental picture of the number system in their heads to use for calculations.</p> <p>Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.</p>	<p>Children use concrete objects to make and count equal groups of objects.</p>  <p>They will count on in twos using a bead string and number line.</p> <p>They understand doubling as repeated addition.</p> $2 + 2 = 4$ <p>They use concrete and pictorial representation to record their calculations.</p> <p>Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.</p> 	<p>Children use concrete objects to count and share equally into 2 groups.</p> <p>6 cakes shared between 2 people each person gets 3 cakes. <math>6 \div 2 = 3</math></p>  <p>They count a set of objects and halve them by making two equal groups.</p> <p>They understand sharing and halving as dividing by 2.</p> <p>They will begin to use objects to make groups of 2 from a given amount.</p> <p>They use concrete and pictorial representation to record their calculations.</p>  <p>Higher attaining children may be able to represent their calculations using symbols and numbers within a written calculation.</p>

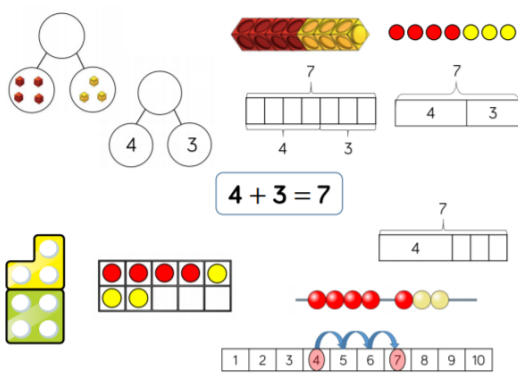
# Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'

YEAR 1 Addition			
Objective / Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	 <p>Use part, part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	$8 = 5 + 3$ $5 + 3 = 8$  <p>Use the part part whole diagram as shown above to move into the abstract.</p> <p>Include missing number questions to support varied fluency:</p> $8 = ? + 3$ $5 + ? = 8$
Starting at the bigger number and counting on	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	 $12 + 5 = 17$ <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	$5 + 12 = 17$ <p>Place the larger number in your head and count on the smaller number to find your answer.</p>
Regrouping to make 10. <i>This is an essential skill for column addition later.</i>	 <p>Start with the bigger number and use the smaller number to make 10.</p> <p>Use ten frames.</p> $6 + 5 = 11$	 <p>Use pictures or a number line. Regroup or partition the smaller number using the part, part whole model to make 10.</p> $9 + 5 = 14$	$7 + 4 = 11$ <p>If I am at seven, how many more do I need to make 10? How many more do I add on now?</p>
Represent & use number bonds and related subtraction facts within 20	 <p>2 more than 5.</p>	 <p>5 + 2 =</p>	<p>Include missing number questions:</p> $8 = ? + 3$ $5 + ? = 8$ <p>Emphasis should be on the language          '1 more than 5 is equal to 6.'          '2 more than 5 is 7.'          '8 is 3 more than 5.'</p>

**Skill: Add 1-digit numbers within 10**

**Year: 1**



When adding numbers to 10, children can explore both aggregation and augmentation.

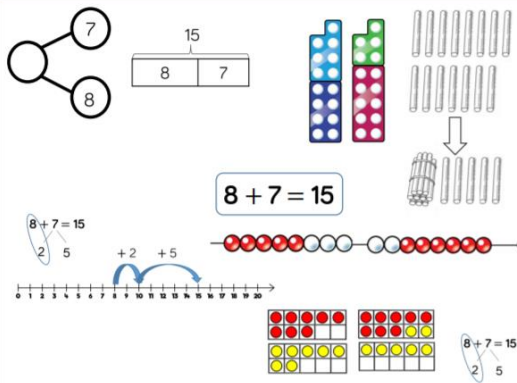
The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation.

The combination bar model, ten frame, bead string and number track all support augmentation.

 $4 + 3 = 7$ 

**Skill: Add 1 and 2-digit numbers to 20**

**Year: 1/2**



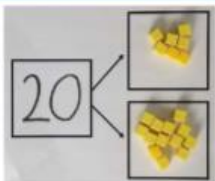
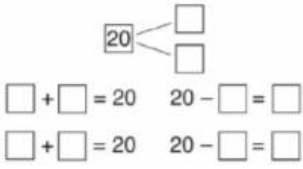
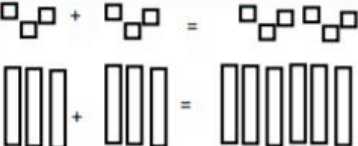
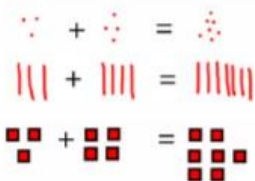



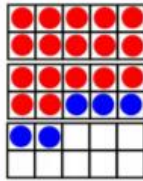
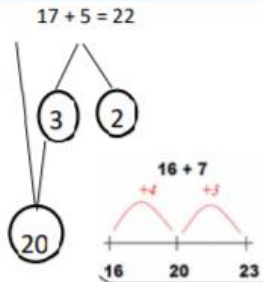
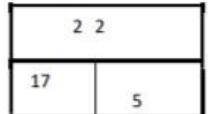



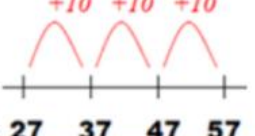

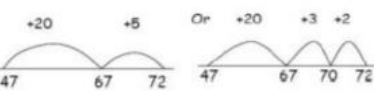

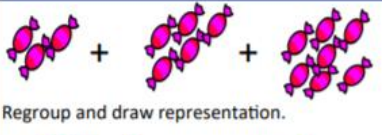
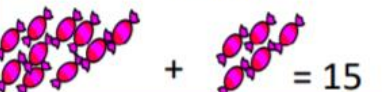
When adding one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.

Different manipulatives can be used to represent this exchange. Use concrete resources alongside number lines to support children in understanding how to partition their jumps.

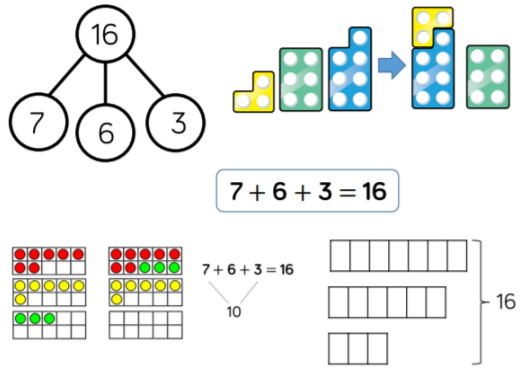
 $8 + 7 = 15$

# YEAR 2 Addition

Objective /Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$  Model using dienes and bead strings	 $3 \text{ tens} + 5 \text{ tens} = \text{ } \text{ tens}$ $30 + 50 = \text{ } \text{ }$ Use representations for base ten.	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts <i>Part, part whole</i>	 Children explore ways of making numbers within 20	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations. $\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts		 Children draw representations of H, T and O	$3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$
Add a two digit number and ones	 $17 + 5 = 22$ Use ten frame to make 'magic ten' Children explore the pattern. $17 + 5 = 22$ $27 + 5 = 32$	Use part part whole and number line to model. $17 + 5 = 22$  $16 + 7 = 23$	$17 + 5 = 22$ Explore related facts $17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$ $22 - 5 = 17$  Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.

<p>Add a 2 digit number and tens</p>	 <p><math>25 + 10 = 35</math></p> <p>Explore that the ones digit does not change</p>	<p><math>27 + 30</math></p> <p><math>+10 +10 +10</math></p>  <p><math>27 \quad 37 \quad 47 \quad 57</math></p>	<p><math>27 + 10 = 37</math></p> <p><math>27 + 20 = 47</math></p> <p><math>27 + \square = 57</math></p>
<p>Add two 2-digit numbers</p>	 <p>Model using dienes, place value counters and numicon</p>	<p><math>+20 +5</math> Or <math>+20 +3 +2</math></p>  <p>Use number line and bridge ten using part whole if necessary.</p>	<p><math>25 + 47</math></p> <p><math>20 + 5 = 25</math>   <math>40 + 7 = 47</math></p> <p><math>20 + 40 = 60</math></p> <p><math>5 + 7 = 12</math></p> <p><math>60 + 12 = 72</math></p>
<p>Add three 1-digit numbers</p>	 <p>Combine to make 10 first if possible, or bridge 10 then add third digit</p>	 <p>Regroup and draw representation.</p>  <p><math>= 15</math></p>	<p><math>4 + 7 + 6 = 10 + 7</math></p> <p><math>= 17</math></p> <p>Combine the two numbers that make/bridge ten then add on the third.</p>

**Skill: Add three 1-digit numbers**      **Year: 2**



$7 + 6 + 3 = 16$

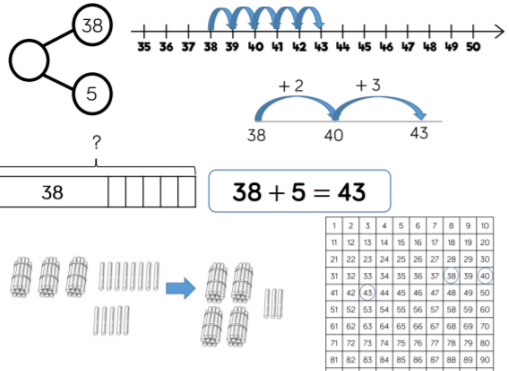
$7 + 6 + 3 = 16$

When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently.

This supports children in their understanding of commutativity.

Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

**Skill: Add 1-digit and 2-digit numbers to 100**      **Year: 2/3**



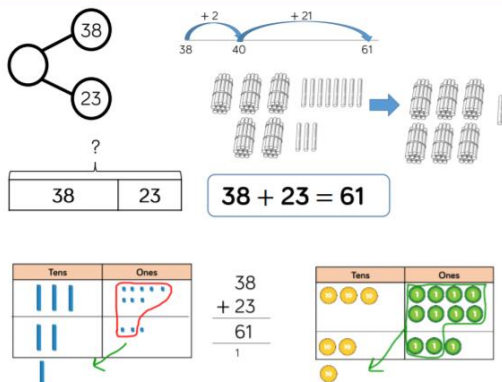
$38 + 5 = 43$

When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

They should also apply their knowledge of number bonds to add more efficiently e.g.  $8 + 5 = 13$  so  $38 + 5 = 43$ .

Hundred squares and straws can support children to find the number bond to 10.

**Skill: Add two 2-digit numbers to 100**      **Year: 2/3**



$38 + 23 = 61$

$38 + 23 = 61$

At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

Children can also use a blank number line to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient.

# YEAR 3 Addition

Objective /Strategy	Concrete	Pictorial	Abstract
<p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3digit numbers.</p>	<p>Dienes or numicon</p> <p>Add together the ones first, then the tens.</p> <p>21 + 42 = + 42</p> <p>Move to using place value counters</p>	<p>Children move to drawing the counters using a tens and one frame.</p>	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>
<p>Column Addition with regrouping.</p>	<p>Exchange ten ones for a ten. Model using numicon and pv counters.</p> <p>Calculations</p> $\begin{array}{r} 146 \\ + 527 \\ \hline \end{array}$	<p>Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line</p>	$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ <p>Start by partitioning the numbers before formal column to show the exchange.</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$
<p>Estimate the answers to questions and use inverse operations to check answers</p>	<p>Estimating <math>98 + 17 = ?</math> <math>100 + 20 = 120</math></p>	<p>Use number lines to illustrate estimation.</p>	<p>Building up known facts and using them to illustrate the inverse and to check answers:</p> $98 + 18 = 116 \quad 116 - 18 = 98$ $18 + 98 = 116 \quad 116 - 98 = 18$

**Skill: Add two 2-digit numbers to 100** Year: 2/3

At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

Children can also use a blank number line to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient.

$$38 + 23 = 61$$

**Skill: Add numbers with up to 3 digits** Year: 3

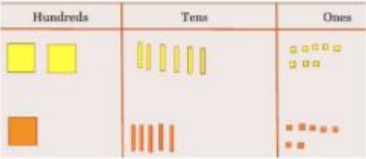
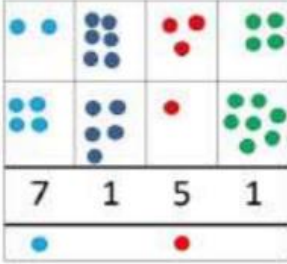
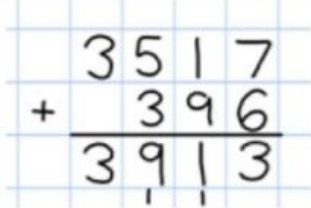
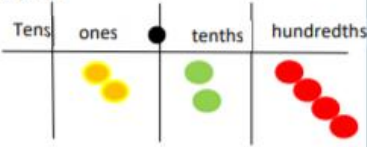
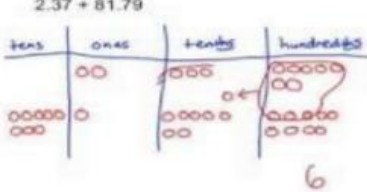
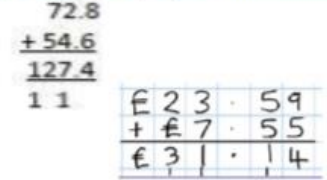
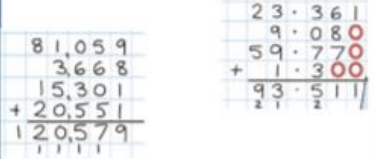
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

$$265 + 164 = 429$$

# YEARS 4 – 6 Addition

Objective /Strategy	Concrete	Pictorial	Abstract
<b>Years 4 – 6</b> Estimate and use inverse operations to check answers to a calculation	<b>AS per Year 3</b>		
Y4—add numbers with up to 4 digits	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 <p>Draw representations using place value grid.</p>	 <p>Continue from previous work to carry hundreds as well as tens. Relate to money and measures.</p>
Y5—add numbers with more than 4 digits.  Add decimals with 2 decimal places, including money.	As year 4  <p>Introduce decimal place value counters and model exchange for addition.</p>		
Y6—add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	Insert zeros for place holders. 

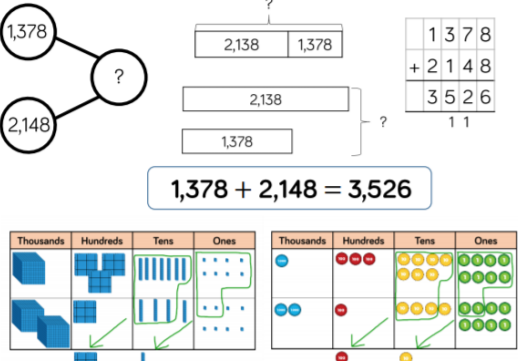
**Skill: Add numbers with up to 4 digits**

**Year: 4**

Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.



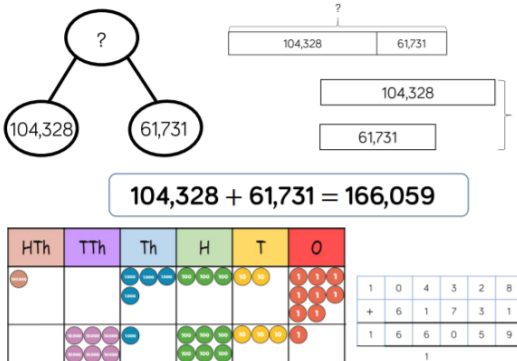
$1,378 + 2,148 = 3,526$

**Skill: Add numbers with more than 4 digits**

**Year: 5/6**

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.



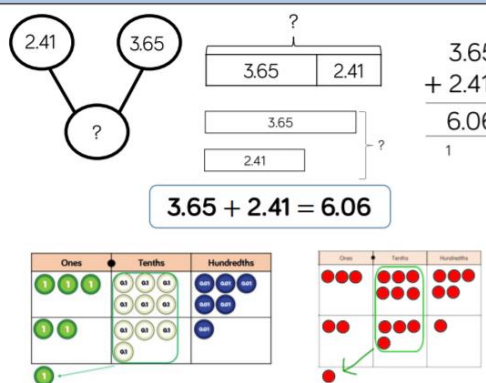
$104,328 + 61,731 = 166,059$

**Skill: Add with up to 3 decimal places**

**Year: 5**

Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places.



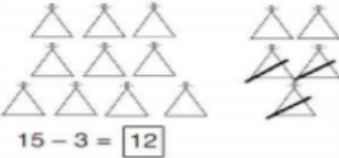

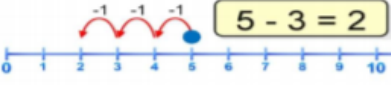
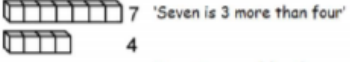
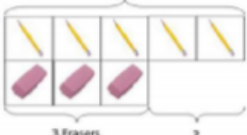
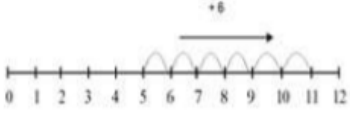
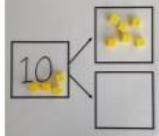
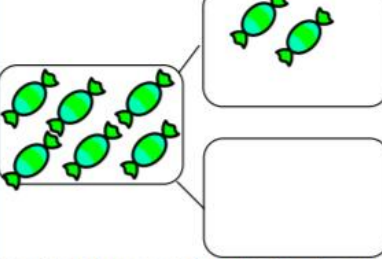
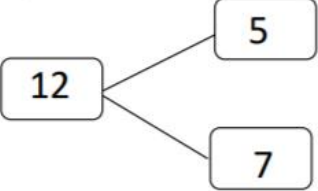


Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

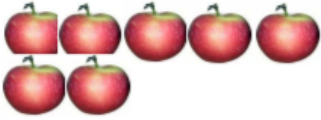
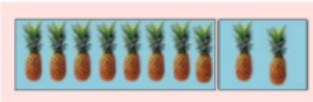


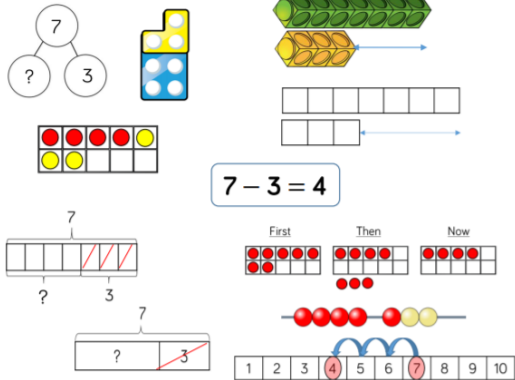
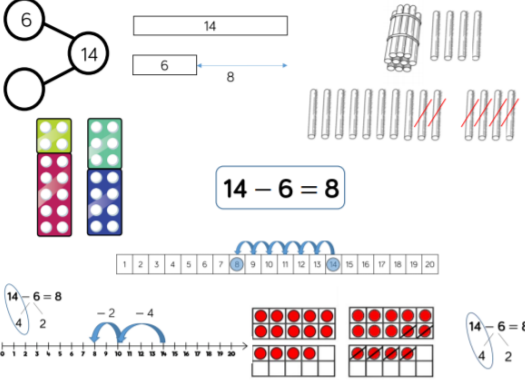
$3.65 + 2.41 = 6.06$

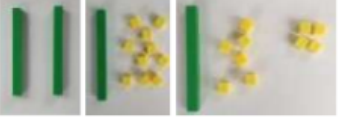
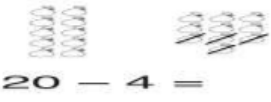
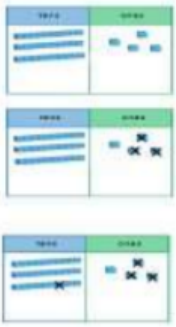

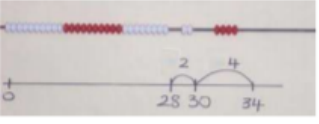
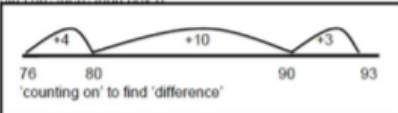
# Subtraction

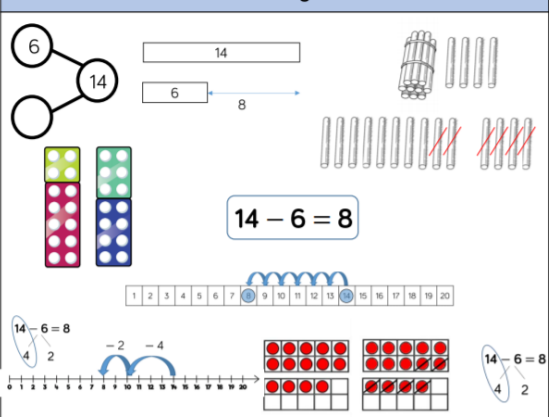
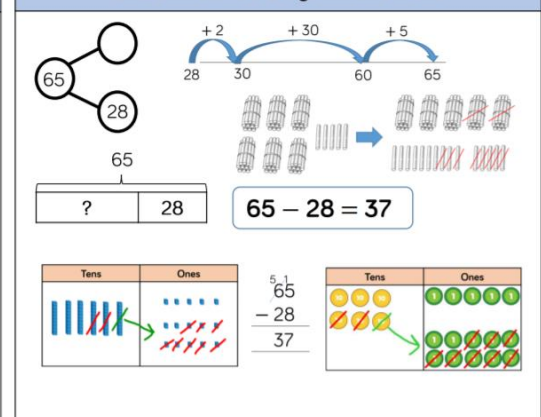
Key language: take away, less than, the difference, minus, subtract, fewer, decrease.


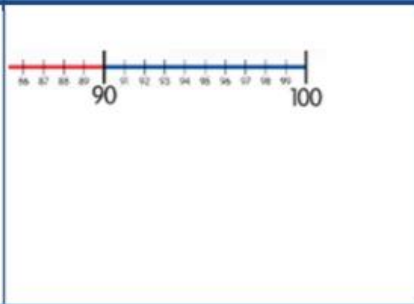
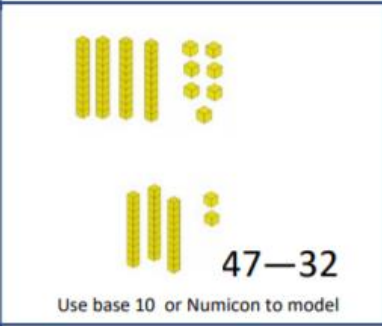
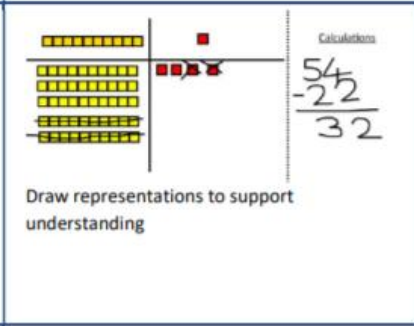
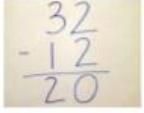
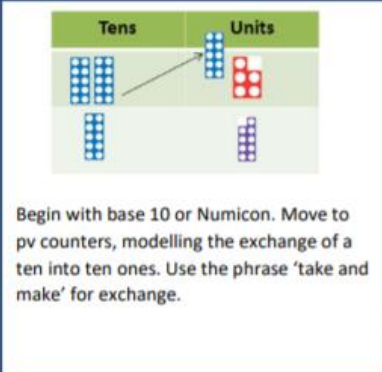
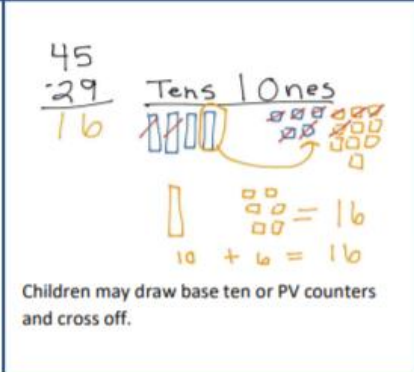
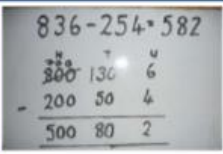

YEAR 1 SUBTRACTION			
Objective /Strategy	Concrete	Pictorial	Abstract
Taking away ones.	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p> <p><math>4 - 2 = 2</math></p>  <p><math>6 - 4 = 2</math></p> 	<p>Cross out drawn objects to show what has been taken away.</p>  <p><math>15 - 3 = 12</math></p>	<p><math>7 - 4 = 3</math></p> <p><math>16 - 9 = 7</math></p>
Counting back	 <p>Move objects away from the group, counting backwards.</p>  <p><math>5 - 3 = 2</math></p> <p>Count back in ones using a number line.</p>	<p>Put 13 in your head, count back 4. What number are you at?</p>	
Find the Difference	<p>Compare objects and amounts</p>  <p><math>7</math> 'Seven is 3 more than four'</p> <p><math>4</math></p> <p>'I am 2 years older than my sister'</p>  <p>5 Pencils</p> <p>3 Erasers</p> <p>7</p> <p>Lay objects to represent bar model.</p>	<p>Count on using a number line to find the difference.</p>  <p><math>+6</math></p>	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?</p>
Represent and use number bonds and related subtraction facts within 20	 <p>Link to addition. Use PPW model to model the inverse.</p> <p>If 10 is the whole and 6 is one of the parts, what's the other part?</p> <p><math>10 - 6 = 4</math></p>	 <p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part whole model.</p> 
Make 10	<p><math>14 - 9</math></p>  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	<p><math>13 - 7</math></p>  <p><math>13 - 7 = 6</math></p> <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>	<p><math>16 - 8</math></p> <p>How many do we take off first to get to 10? How many left to take off?</p>

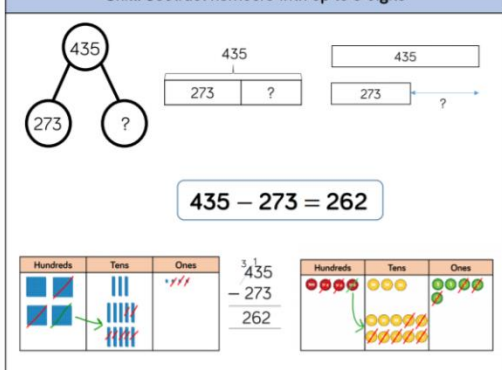
<p>Bar model</p> <p>Including the inverse operations.</p>	 <p><math>5 - 2 = 3</math></p>		<table border="1" style="width: 100%; text-align: center;"> <tr> <td style="width: 50%; padding: 10px;">8</td> <td style="width: 50%; padding: 10px;">2</td> </tr> </table> <p style="text-align: center;"><math>10 = 8 + 2</math></p> <p style="text-align: center;"><math>10 = 2 + 8</math></p> <p style="text-align: center;"><math>10 - 2 = 8</math></p> <p style="text-align: center;"><math>10 - 8 = 2</math></p>	8	2
8	2				

Skill: Subtract 1-digit numbers within 10	Year: 1	Skill: Subtract 1 and 2-digit numbers to 20	Year: 1/2
 <p><math>7 - 3 = 4</math></p>	<p>Part-whole models, bar models, ten frames and number shapes support partitioning.</p> <p>Ten frames, number tracks, single bar models and bead strings support reduction.</p> <p>Cubes and bar models with two bars can support finding the difference.</p>	 <p><math>14 - 6 = 8</math></p>	<p>When subtracting one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.</p> <p>Children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.</p>

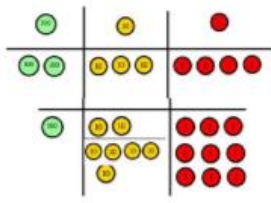
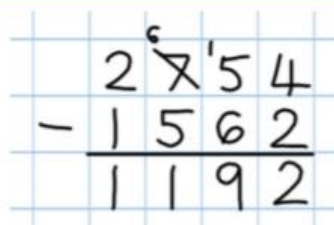
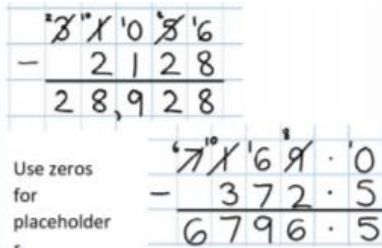
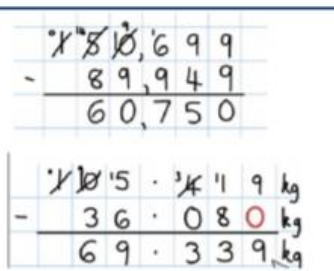
YEAR 2 - SUBTRACTION			
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Regroup a ten into ten ones</p>	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>	 <p><math>20 - 4 = 16</math></p>	<p><math>20 - 4 = 16</math></p>
<p>Partitioning to subtract without regrouping.</p> <p>'Friendly numbers'</p>	<p><math>34 - 13 = 21</math></p>  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	<p>Children draw representations of Dienes and cross off.</p>  <p><math>43 - 21 = 22</math></p>	<p><math>43 - 21 = 22</math></p>
<p>Make ten strategy</p> <p>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</p>	 <p><math>34 - 28 = 6</math></p> <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>'counting on' to find 'difference'</p> <p>Use a number line to count on to next ten and then the rest.</p>	<p><math>93 - 76 = 17</math></p>

Skill: Subtract 1 and 2-digit numbers to 20	Year: 1/2	Skill: Subtract 1 and 2-digit numbers to 100	Year: 2
 <p>When subtracting one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.</p> <p>Children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.</p>		 <p>At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.</p> <p>Children can also use a blank number line to count on to find the difference. Encourage them to jump to multiples of 10 to become more efficient.</p>	

YEAR 3 - SUBTRACTION			
Objective/ Strategy	Concrete	Pictorial	Abstract
Subtract numbers mentally, including: three digit number + ones three digit number + tens three digit number + hundreds			Vary the position of the answer and question.  Expose children to missing number questions and vary the missing part of the calculation.  $678 = ? - 1$ $688 - 10 = ?$ $678 = ? - 100$
Column subtraction without regrouping (friendly numbers)	 <p>Use base 10 or Numicon to model</p>	 <p>Draw representations to support understanding</p>	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$  Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	 <p>Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.</p>	 <p>Children may draw base ten or PV counters and cross off.</p>	 <p>Begin by partitioning into pv columns</p>   <p>Then move to formal method.</p>

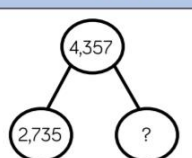
Skill: Subtract numbers with up to 3 digits	Year: 3
 <p>Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.</p> <p>Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.</p> <p>Plain counters on a place value grid can also be used to support learning.</p>	

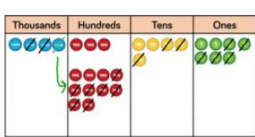
# YEARS 4 – 6 SUBTRACTION

Objective /Strategy	Concrete	Pictorial	Abstract
<p><b>Subtracting tens and ones</b></p> <p>Year 4 subtract with up to 4 digits.</p> <p><i>Introduce decimal subtraction through context of money</i></p>	<p><b>234 - 179</b></p>  <p>Model process of exchange using Numicon, base ten and then move to PV counters.</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use the phrase 'take and make' for exchange</p>
<p><b>Year 5- Subtract with at least 4 digits, including money and measures.</b></p> <p><i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal Up to 3 decimal places</i></p>	<p>As Year 4</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use zeros for placeholder s.</p>
<p><b>Year 6—Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).</b></p>	<p>As Year 4</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	

**Skill: Subtract numbers with up to 4 digits**

**Year: 4**





$$4,357 - 2,735 = 1,622$$

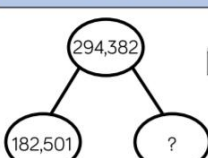
Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits.

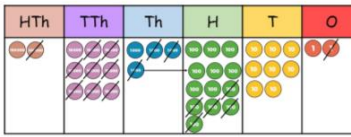
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

**Skill: Subtract numbers with more than 4 digits**

**Year: 5/6**





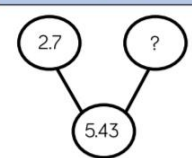
$$294,382 - 182,501 = 111,881$$

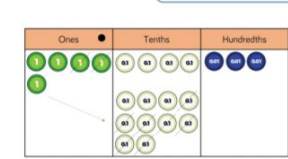
Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.

**Skill: Subtract with up to 3 decimal places**

**Year: 5**





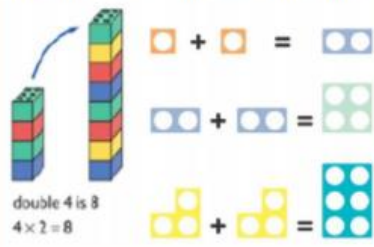

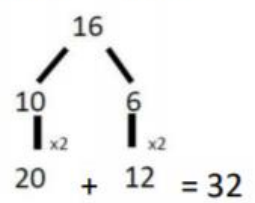

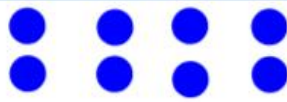
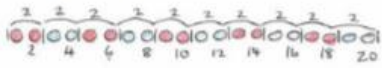


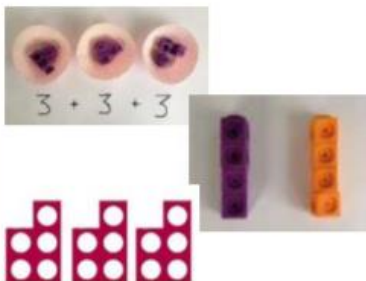
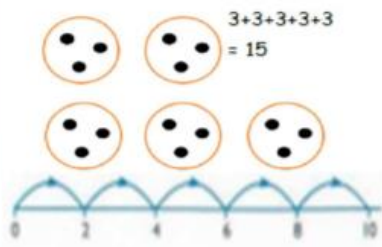

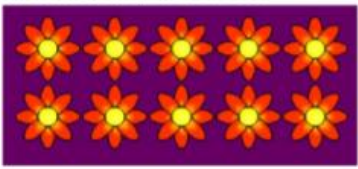
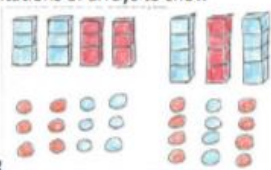
$$5.43 - 2.7 = 2.73$$

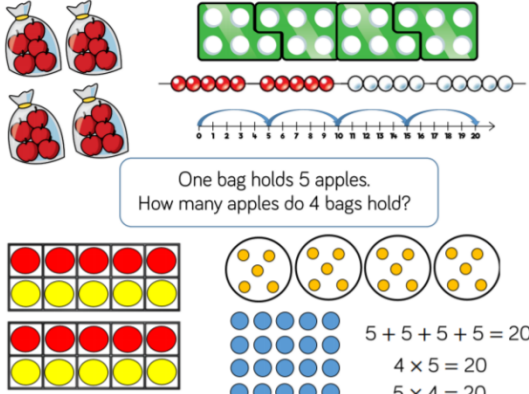
Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

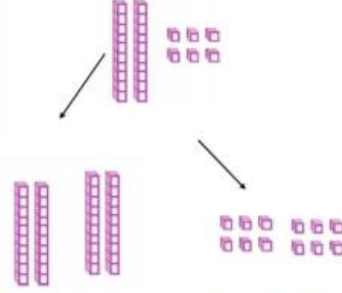
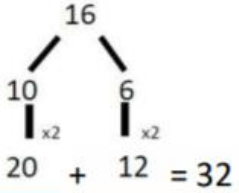
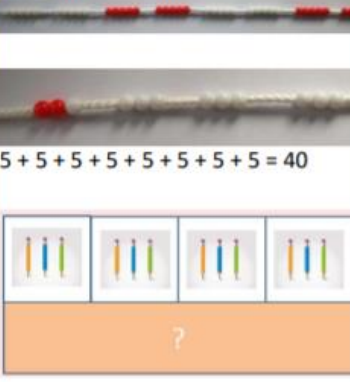
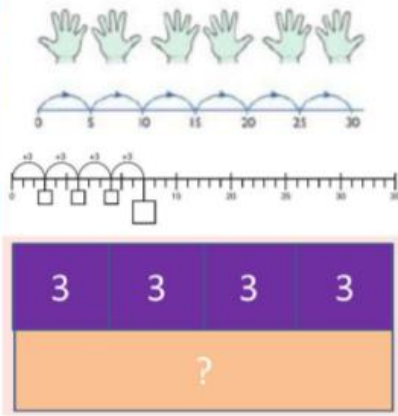
Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.


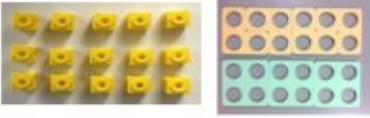

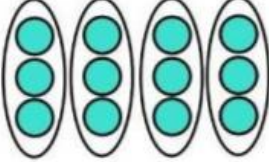
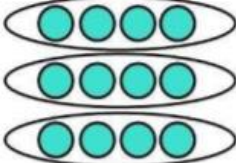


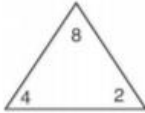
# Multiplication

Key language: Double, times, multiplied by, the product of, groups of, lots of, equal groups.

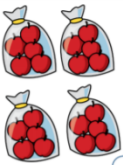
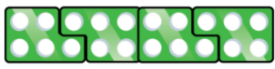

YEAR 1 MULTIPLICATION			
Programme of Study specifies the following objectives, however it does not require the explicit teaching of the mathematical symbol of multiplication			
Objective / Strategy	Concrete	Pictorial	Abstract
<b>Doubling</b>	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling  <p>double 4 is 8 <math>4 \times 2 = 8</math></p>	Draw pictures to show how to double numbers <p>Double 4 is 8</p> 	Partition a number and then double each part before recombining it back together. 
<b>Counting in multiples (2s, 5s, 10s)</b>	Count the groups as children are skip counting, children may use their fingers as they are skip counting. 	 <p>Children make representations to show counting in multiples.</p> 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
<b>Making equal groups and counting the total</b>	 <p>Use manipulatives to create equal groups.</p> <p><math>\square \times \square = 8</math></p>	Draw  to show $2 \times 3 = 6$ Draw and make representations	$2 \times 4 = 8$
<b>Repeated addition</b>	 <p>Use different objects to add equal groups</p>	Use pictorial including number lines to solve prob. There are 3 sweets in one bag. How many sweets are in 5 bags altogether? $3 + 3 + 3 + 3 + 3 = 15$ 	Write addition sentences to describe objects and pictures.  <p><math>2 + 2 + 2 + 2 + 2 = 10</math></p>
<b>Understanding arrays</b>	Use objects laid out in arrays to find the answers to 2 lots of 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	$3 \times 2 = 6$ $2 \times 5 = 10$

Skill: Solve 1-step problems using multiplication	Year: 1/2
 <p data-bbox="566 302 869 358">One bag holds 5 apples. How many apples do 4 bags hold?</p> <p data-bbox="805 448 973 537"> <math>5 + 5 + 5 + 5 = 20</math>  <math>4 \times 5 = 20</math>  <math>5 \times 4 = 20</math> </p>	<p data-bbox="997 145 1145 235">Children represent multiplication as repeated addition in many different ways.</p> <p data-bbox="997 257 1145 403">In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally.</p> <p data-bbox="997 425 1145 492">In Year 2, children are introduced to the multiplication symbol.</p>

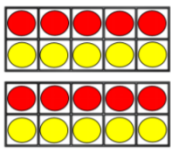
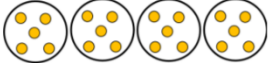
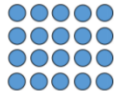
YEAR 2 MULTIPLICATION			
Children should be able to recall and use multiplication and division facts for the 2, 5 and 10 times tables.			
Objective / Strategy	Concrete	Pictorial	Abstract
<p data-bbox="87 824 295 851">Doubling</p>	<p data-bbox="306 824 660 884">Model doubling using dienes and PV counters.</p>  <p data-bbox="359 1198 630 1243"><math>40 + 12 = 52</math></p>	<p data-bbox="673 824 1075 884">Draw pictures and representations to show how to double numbers</p>	<p data-bbox="1088 824 1509 884">Partition a number and then double each part before recombining it back together.</p>  <p data-bbox="1189 1198 1428 1243"><math>20 + 12 = 32</math></p>
<p data-bbox="87 1256 295 1400">Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)</p>	<p data-bbox="306 1256 660 1400">Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.</p>  <p data-bbox="306 1545 622 1579"><math>5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40</math></p>	<p data-bbox="673 1256 1075 1355">Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p> 	<p data-bbox="1088 1256 1509 1288">Count in multiples of a number aloud.</p> <p data-bbox="1088 1344 1509 1400">Write sequences with multiples of numbers.</p> <p data-bbox="1088 1422 1236 1444">0, 2, 4, 6, 8, 10</p> <p data-bbox="1088 1467 1252 1489">0, 3, 6, 9, 12, 15</p> <p data-bbox="1088 1512 1316 1534">0, 5, 10, 15, 20, 25, 30</p> <p data-bbox="1088 1635 1372 1713"><math>4 \times 3 = \square</math></p>

<p><b>Multiplication is commutative</b></p>	<p>Create arrays using counters and cubes and Numicon.</p>   <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p> 	<p>Use representations of arrays to show different calculations and explore commutativity.</p>  	<p><math>12 = 3 \times 4</math> <math>12 = 4 \times 3</math></p> <p>3</p> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p><math>5 + 5 + 5 = 15</math>  <math>3 + 3 + 3 + 3 + 3 = 15</math>  <math>5 \times 3 = 15</math>  <math>3 \times 5 = 15</math></p>
<p><b>Using the Inverse</b></p> <p><i>This should be taught alongside division, so pupils learn how they work alongside each other.</i></p>		 <p>□ × □ = □  □ × □ = □  □ ÷ □ = □  □ ÷ □ = □</p>	<p><math>2 \times 4 = 8</math>  <math>4 \times 2 = 8</math>  <math>8 \div 2 = 4</math>  <math>8 \div 4 = 2</math>  <math>8 = 2 \times 4</math>  <math>8 = 4 \times 2</math>  <math>2 = 8 \div 4</math>  <math>4 = 8 \div 2</math></p> <p>Show all 8 related fact family sentences.</p>

**Skill: Solve 1-step problems using multiplication** **Year: 1/2**

One bag holds 5 apples.  
How many apples do 4 bags hold?


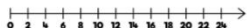


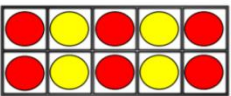

$5 + 5 + 5 + 5 = 20$   
 $4 \times 5 = 20$   
 $5 \times 4 = 20$

Children represent multiplication as repeated addition in many different ways.

In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally.

In Year 2, children are introduced to the multiplication symbol.

**Skill: 2 times table** **Year: 2**


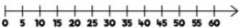


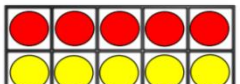
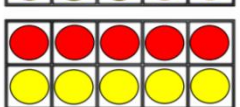








Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the two times table, using concrete manipulatives to support. Notice how all the numbers are even and there is a pattern in the ones.

Use different models to develop fluency.


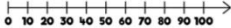

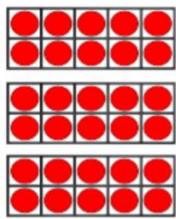


**Skill: 5 times table** **Year: 2**

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the five times table, using concrete manipulatives to support. Notice the pattern in the ones as well as highlighting the odd, even, odd, even pattern.

**Skill: 10 times table** **Year: 2**

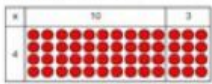
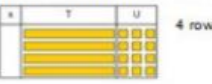
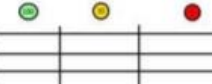
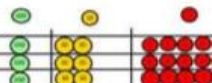
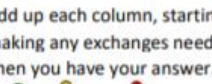
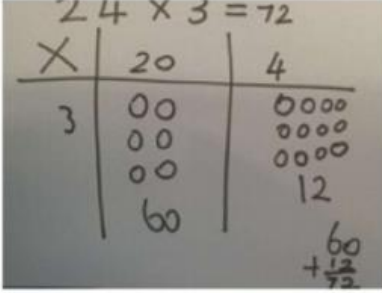
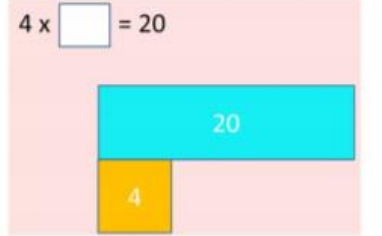







Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

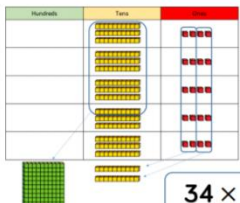
Look for patterns in the ten times table, using concrete manipulatives to support. Notice the pattern in the digits—the ones are always 0, and the tens increase by 1 ten each time.

# YEAR 3 MULTIPLICATION

Children should be able to recall and use multiplication facts for the 3, 4, and 8 times tables

Objective /Strategy	Concrete	Pictorial	Abstract															
<p>Grid method, progressing to the formal method</p> <p>Multiply 2 digit numbers by 1 digit numbers</p>	<p>Show the links with arrays to first introduce the grid method.</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move onto base ten to move towards a more compact method.</p>  <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations 4 x 126</p>  <p>Calculations 4 x 126</p> <p>Fill each row with 126. Add up each column, starting with the ones making any exchanges needed Then you have your answer.</p> 	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p>  <p>Bar model are used to explore missing numbers</p>  <p>4 x <input type="text"/> = 20</p>	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1181 358 1436 436"> <tr> <td>X</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p><math>210 + 35 = 245</math></p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table border="1" data-bbox="1181 660 1436 817"> <tr> <td></td> <td>10</td> <td>8</td> </tr> <tr> <td>10</td> <td>100</td> <td>80</td> </tr> <tr> <td>3</td> <td>30</td> <td>24</td> </tr> </table> <p>Move forward to the formal written method:</p> $\begin{array}{r} 35 \\ \times 7 \\ \hline 245 \\ \hline 3 \end{array}$	X	30	5	7	210	35		10	8	10	100	80	3	30	24
X	30	5																
7	210	35																
	10	8																
10	100	80																
3	30	24																

**Skill: Multiply 2-digit numbers by 1-digit numbers** **Year: 3/4**

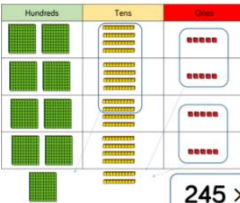


$34 \times 5 = 170$

	H	T	O
x		3	4
		2	0
	1	7	0

Teachers may decide to first look at the expanded column method before moving on to the short multiplication method. The place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

**Skill: Multiply 3-digit numbers by 1-digit numbers** **Year: 3/4**

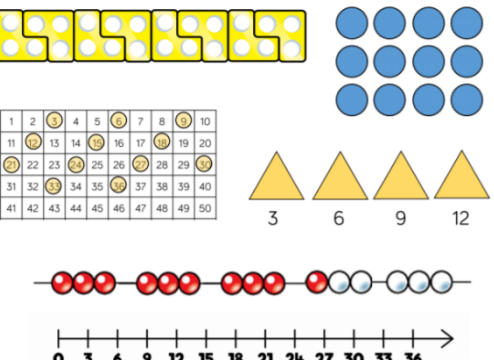


$245 \times 4 = 980$

	H	T	O
x	2	4	5
	9	8	0

When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method. Base 10 and place value counters continue to support the understanding of the written method. Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

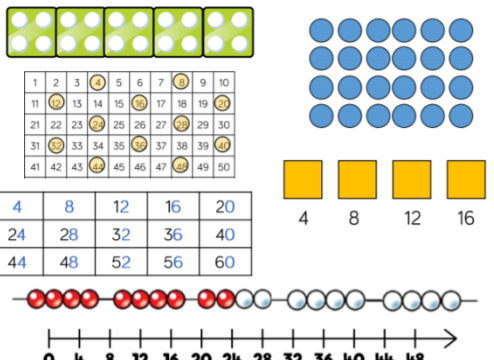
**Skill: 3 times table** **Year: 3**



Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the three times table, using concrete manipulatives to support. Notice the odd, even, odd, even pattern using number shapes to support. Highlight the pattern in the ones using a hundred square.

**Skill: 4 times table** **Year: 3**



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the four times table, using manipulatives to support. Make links to the 2 times table, seeing how each multiple is double the twos. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

**Skill: 8 times table** **Year: 3**

8	16	24	32	40
48	56	64	72	80

Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the eight times table, using manipulatives to support. Make links to the 4 times table, seeing how each multiple is double the fours. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

## YEAR 4 MULTIPLICATION

Pupils should be able to recall and use multiplication facts for the 6, 9, 7, 11 and 12 times tables.

Objective /Strategy	Concrete	Pictorial	Abstract																							
<p>Grid method recap from year 3 for 2 digits x 1 digit</p> <p>Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)</p>	<p>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p> <p>Calculations <math>4 \times 126</math></p> <p>Fill each row with 126</p> <p>Add up each column making any exchanges needed</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p>	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" style="margin: 10px auto;"> <tr><td><b>x</b></td><td><b>30</b></td><td><b>5</b></td></tr> <tr><td><b>7</b></td><td><b>210</b></td><td><b>35</b></td></tr> </table> <p style="text-align: center;"><math>210 + 35 = 245</math></p>	<b>x</b>	<b>30</b>	<b>5</b>	<b>7</b>	<b>210</b>	<b>35</b>																	
<b>x</b>	<b>30</b>	<b>5</b>																								
<b>7</b>	<b>210</b>	<b>35</b>																								
<p>Column multiplication</p>	<p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. <math>321 \times 2 = 642</math></p> <table border="1" style="margin: 10px auto;"> <tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr> <tr><td>3</td><td>2</td><td>1</td></tr> <tr><td>6</td><td>4</td><td>2</td></tr> </table> <p>It is important at this stage that they always multiply the ones first.</p> <p>The corresponding long multiplication is modelled alongside</p>	Hundreds	Tens	Ones	3	2	1	6	4	2	<p>The grid method may be used to show how this relates to a formal written method.</p> <p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p>	<div style="text-align: right;"> <math display="block">\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}</math> </div> <p style="text-align: right;">This may lead to a compact method.</p> <div style="text-align: right;"> <table border="1" style="margin: 10px auto;"> <tr><td>3</td><td>2</td><td>7</td></tr> <tr><td>x</td><td></td><td>4</td></tr> <tr><td>1</td><td>3</td><td>0</td><td>8</td></tr> <tr><td></td><td>1</td><td>2</td><td></td></tr> </table> </div>	3	2	7	x		4	1	3	0	8		1	2	
Hundreds	Tens	Ones																								
3	2	1																								
6	4	2																								
3	2	7																								
x		4																								
1	3	0	8																							
	1	2																								

**Skill: Multiply 2-digit numbers by 1-digit numbers**

**Year: 3/4**

Teachers may decide to first look at the expanded column method before moving on to the short multiplication method. The place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

$34 \times 5 = 170$

**Skill: Multiply 3-digit numbers by 1-digit numbers**

**Year: 3/4**

When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method. Base 10 and place value counters continue to support the understanding of the written method. Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

$245 \times 4 = 980$

**Skill: 6 times table**

**Year: 4**

Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the six times table, using manipulatives to support. Make links to the 3 times table, seeing how each multiple is double the threes. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

6	12	18	24	30
36	42	48	54	60
66	72	78	84	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**Skill: 9 times table**

**Year: 4**

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the nine times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support as well as noting the odd, even pattern within the multiples.

9	18	27	36	45
54	63	72	81	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**Skill: 7 times table**

**Year: 4**

Encourage daily counting in multiples both forwards and backwards, supported by a number line or a hundred square. The seven times table can be trickier to learn due to the lack of obvious pattern in the numbers, however they already know several facts due to commutativity. Children can still see the odd, even pattern in the multiples using number shapes to support.

7	14	21	28	35
42	49	56	63	70

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**Skill: 11 times table**

**Year: 4**

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the eleven times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support. Also consider the pattern after crossing 100.

11	22	33	44	55	66
77	88	99	110	121	132

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**Skill: 12 times table**

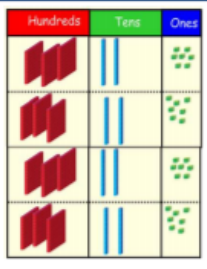
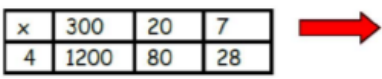
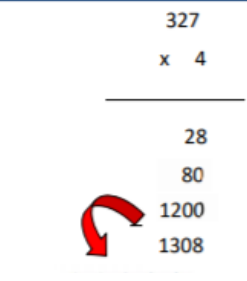
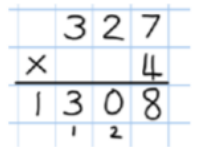
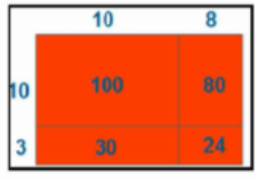
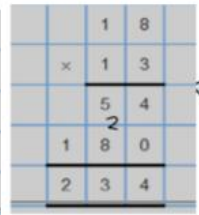
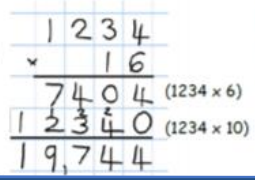
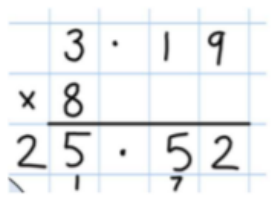
**Year: 4**

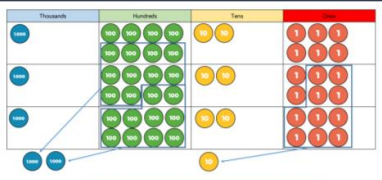
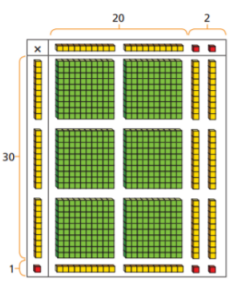
Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the 12 times table, using manipulatives to support. Make links to the 6 times table, seeing how each multiple is double the sixes. Notice the pattern in the ones within each group of five multiples. The hundred square can support in highlighting this pattern.

12	24	36	48	60
72	84	96	108	120
132	144			

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

# YEAR 5/6 MULTIPLICATION

Objective /Strategy	Concrete	Pictorial	Abstract
<p>Column Multiplication for 3 and 4 digits x 1 digit.</p>	 <p>It is important at this stage that they always Multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. <math>321 \times 2 = 642</math></p>		 <p>This will lead to a compact method.</p> 
<p>Column multiplication</p>	<p>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</p>	 <p>Continue to use bar modelling to support problem solving</p>	 <p>18 x 3 on the first row (8 x 3 = 24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first</p> 
<p>Multiplying decimals up to 2 decimal places by a single digit.</p>			<p>Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.</p> 

Skill: Multiply 4-digit numbers by 1-digit numbers	Year: 5	Skill: Multiply 2-digit numbers by 2-digit numbers	Year: 5																																																													
 <p style="text-align: center; border: 1px solid black; border-radius: 10px; padding: 5px;"><math>1,826 \times 3 = 5,478</math></p> <table border="1" style="margin-left: auto; margin-right: auto; text-align: center;"> <tr><td></td><td>Th</td><td>H</td><td>T</td><td>O</td></tr> <tr><td></td><td>1</td><td>8</td><td>2</td><td>6</td></tr> <tr><td>x</td><td></td><td></td><td></td><td>3</td></tr> <tr><td></td><td>5</td><td>4</td><td>7</td><td>8</td></tr> <tr><td></td><td></td><td>2</td><td></td><td>1</td></tr> </table>		Th	H	T	O		1	8	2	6	x				3		5	4	7	8			2		1	<p>When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method. If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.</p>	 <table border="1" style="margin-left: auto; margin-right: auto; text-align: center;"> <tr><td></td><td>x</td><td>20</td><td>2</td></tr> <tr><td>30</td><td></td><td>600</td><td>60</td></tr> <tr><td>1</td><td></td><td>20</td><td>2</td></tr> </table> <p style="text-align: center; border: 1px solid black; border-radius: 10px; padding: 5px;"><math>22 \times 31 = 682</math></p> <table border="1" style="margin-left: auto; margin-right: auto; text-align: center;"> <tr><td></td><td>H</td><td>T</td><td>O</td></tr> <tr><td></td><td></td><td>2</td><td>2</td></tr> <tr><td>x</td><td></td><td>3</td><td>1</td></tr> <tr><td></td><td></td><td>2</td><td>2</td></tr> <tr><td></td><td>6</td><td>6</td><td>0</td></tr> <tr><td></td><td>6</td><td>8</td><td>2</td></tr> </table>		x	20	2	30		600	60	1		20	2		H	T	O			2	2	x		3	1			2	2		6	6	0		6	8	2	<p>When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10. The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.</p>
	Th	H	T	O																																																												
	1	8	2	6																																																												
x				3																																																												
	5	4	7	8																																																												
		2		1																																																												
	x	20	2																																																													
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x		3	1																																																													
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	6	6	0																																																													
	6	8	2																																																													

**Skill: Multiply 3-digit numbers by 2-digit numbers**

**Year: 5**

Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Encourage children to move towards the formal written method, seeing the links with the grid method.

	Th	H	T	O
		2	3	4
x			3	2
	4	6	8	
1	7	1	0	2
7	4	8	8	

x	200	30	4
30	6,000	900	120
2	400	60	8

**$234 \times 32 = 7,488$**

**Skill: Multiply 4-digit numbers by 2-digit numbers**

**Year: 5/6**

When multiplying 4-digits by 2-digits, children should be confident in the written method.

If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.

Consider where exchanged digits are placed and make sure this is consistent.

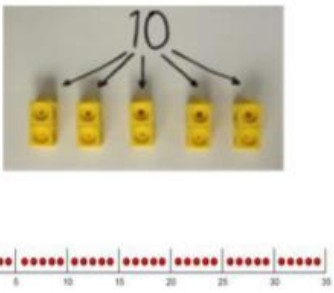
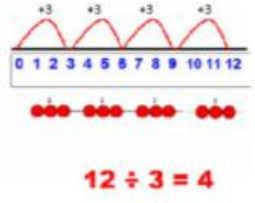
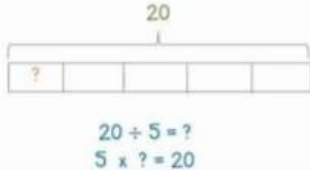
	TTh	Th	H	T	O
		2	7	3	9
x				2	8
	2	1	9	1	2
2	5	3	7		
1	5	4	7	8	0
	7	6	6	9	2


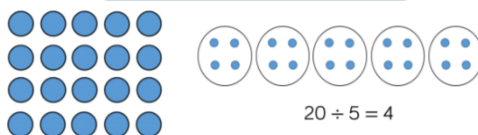
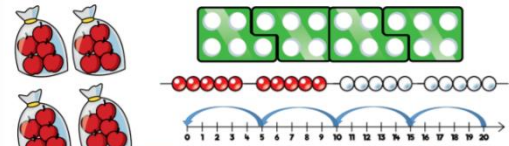
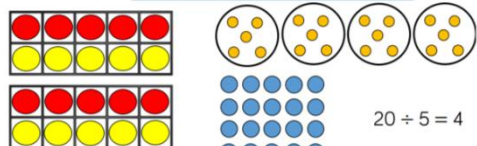
**$2,739 \times 28 = 76,692$**

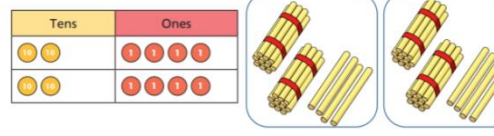
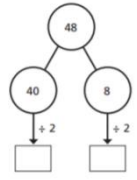
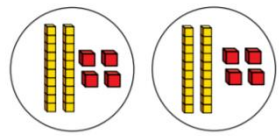
## Division

Key language: Share, group, divide, divided by, half.

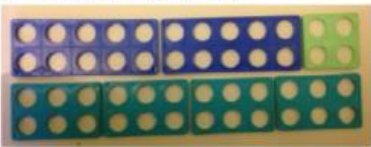
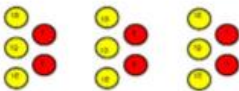


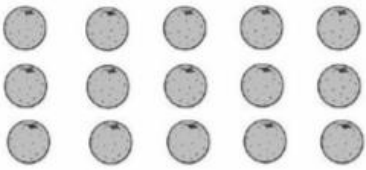
YEAR 1 DIVISION			
Objective /Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p> <p>Use Gordon ITPs for modelling</p>	<p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p> <p>8 shared between 2 is 4</p> <p>Sharing:</p> <p>12 shared between 3 is 4</p>	<p>12 shared between 3 is 4</p>
<p>Division as sharing</p>	<p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p> <p><math>8 \div 2 = 4</math></p> <p>Children use bar modelling to show and support understanding.</p> <p><math>12 \div 4 = 3</math></p>	<p><math>12 \div 3 = 4</math></p>

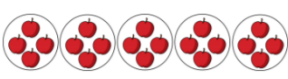
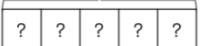
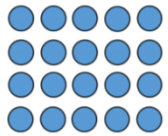

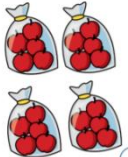
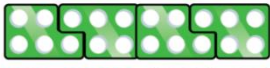
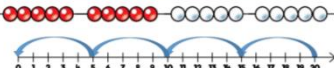

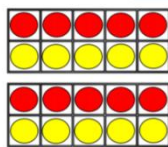
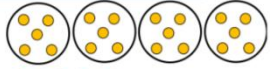
<p><b>Division as grouping</b></p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p> 	<p>Use number lines for grouping</p>  <p><math>12 \div 3 = 4</math></p> <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  <p><math>20 \div 5 = ?</math> <math>5 \times ? = 20</math></p>	<p><math>28 \div 7 = 4</math></p> <p>Divide 28 into 7 groups. How many are in each group?</p>
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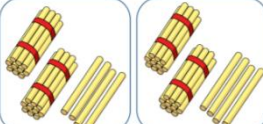
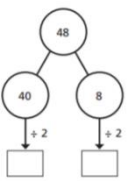
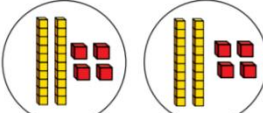








Skill: Solve 1-step problems using multiplication (sharing)	Year: 1/2	Skill: Solve 1-step problems using division (grouping)	Year: 1/2
 <p>There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?</p>  <p><math>20 \div 5 = 4</math></p>	<p>Children solve problems by sharing amounts into equal groups.</p> <p>In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally.</p> <p>In Year 2, children are introduced to the division symbol.</p>	 <p>There are 20 apples altogether. They are put in bags of 5. How many bags are there?</p>  <p><math>20 \div 5 = 4</math></p>	<p>Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.</p>

Skill: Divide 2-digits by 1-digit (sharing with no exchange)	Year: 1/2
 <p><math>48 \div 2 = 24</math></p>  	<p>When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.</p> <p>Straws, Base 10 and place value counters can all be used to share numbers into equal groups.</p> <p>Part-whole models can provide children with a clear written method that matches the concrete representation.</p>

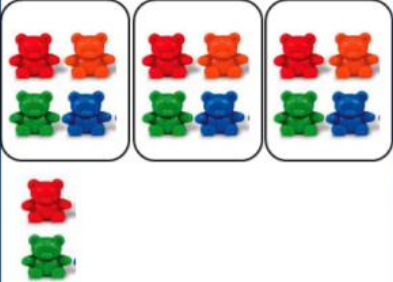
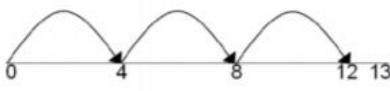
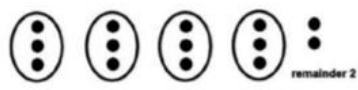
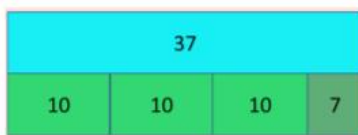
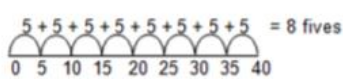
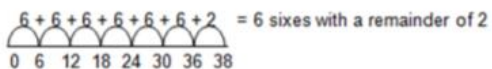
# YEAR 2 DIVISION

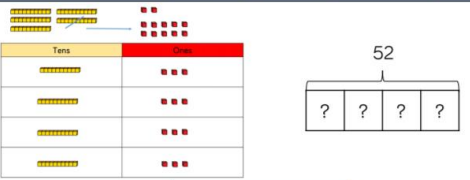
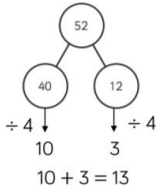
Objective/Strategy	Concrete	Pictorial	Abstract
Division as grouping	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p> <div style="text-align: center;"> <math>20</math>    <math>20 \div 5 = ?</math>  <math>5 \times ? = 20</math> </div>	<p>How many groups of 6 in 24?</p> $24 \div 6 = 4$
Division with arrays	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg <math>15 \div 3 = 5</math>   <math>5 \times 3 = 15</math>  <math>15 \div 5 = 3</math>   <math>3 \times 5 = 15</math></p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences. <math>7 \times 4 = 28</math></p> $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

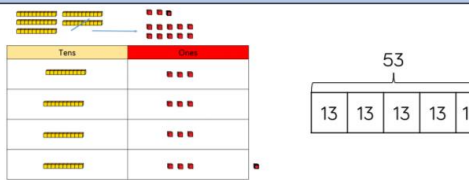
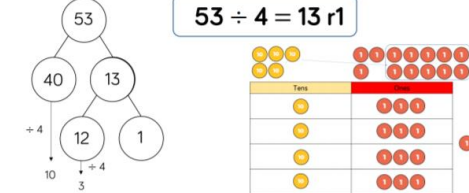
Skill: Solve 1-step problems using multiplication (sharing)	Year: 1/2	Skill: Solve 1-step problems using division (grouping)	Year: 1/2
 <div style="text-align: center;"> <math>20</math>   </div> <p>There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?</p>   $20 \div 5 = 4$	<p>Children solve problems by sharing amounts into equal groups.</p> <p>In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally.</p> <p>In Year 2, children are introduced to the division symbol.</p>	   <div style="text-align: center;"> <math>20</math>   </div> <p>There are 20 apples altogether. They are put in bags of 5. How many bags are there?</p>   $20 \div 5 = 4$	<p>Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.</p>

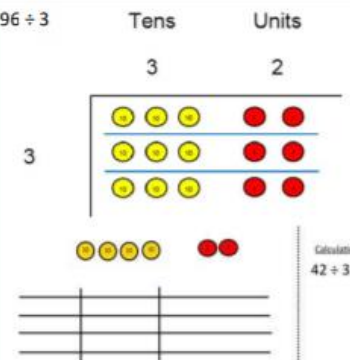
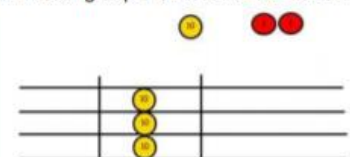
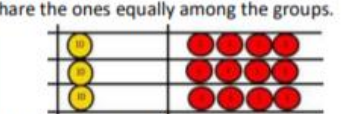
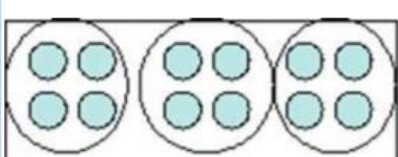
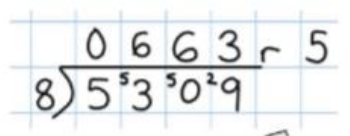
Skill: Divide 2-digits by 1-digit (sharing with no exchange)	Year: 1/2				
<table border="1" style="margin-bottom: 10px;"> <tr> <th style="background-color: #f1c40f;">Tens</th> <th style="background-color: #e74c3c;">Ones</th> </tr> <tr> <td style="text-align: center;">    </td> <td style="text-align: center;">    </td> </tr> </table>  <div style="text-align: center;"> <math>48</math>    <math>48 \div 2 = 24</math> </div> 	Tens	Ones	 	 	<p>When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.</p> <p>Straws, Base 10 and place value counters can all be used to share numbers into equal groups.</p> <p>Part-whole models can provide children with a clear written method that matches the concrete representation.</p>
Tens	Ones				
 	 				

# YEAR 3 DIVISION

Objective /Strategy	Concrete	Pictorial	Abstract
<b>Objective &amp; Strategy</b>  Division with remainders.	$14 \div 3 =$  Divide objects between groups and see how much is left over  	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.    Draw dots and group them to divide an amount and clearly show a remainder.    Use bar models to show division with remainders.  	Complete written divisions and show the remainder using r.  $29 \div 8 = 3 \text{ REMAINDER } 5$ ↑    ↑    ↑                    ↑ dividend    divisor    quotient                    remainder
		<p><b>Example without remainder:</b>  <math>40 \div 5</math>                      Ask "How many 5s in 40?"</p>  <p><b>Example with remainder:</b>  <math>38 \div 6</math></p> 	
		<p>For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.</p>	

Skill: Divide 2-digits by 1-digit (sharing with exchange)	Year: 3/4
  $52 \div 4 = 13$  	<p>When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.</p> <p>Flexible partitioning in a part-whole model supports this method.</p>

Skill: Divide 2-digits by 1-digit (sharing with remainders)	Year: 3/4
  $53 \div 4 = 13 \text{ r}1$  	<p>When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones. Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made. Flexible partitioning in a part-whole model supports this method.</p>

Objective/Strategy	Concrete	Pictorial	Abstract
<p>Divide at least 3 digit numbers by 1 digit.</p> <p>Short Division</p>	<p><math>96 \div 3</math></p> <p>Tens      Units</p> <p>3            2</p>  <p>Calculations <math>42 \div 3</math></p> <p>Use place value counters to divide using the bus stop method alongside</p> <p><math>42 \div 3 =</math></p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder.</p> $\begin{array}{r} 218 \\ 3 \overline{) 654} \end{array}$ <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$ <p>Finally move into decimal places to divide the total accurately.</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$ 

## Long Division

Step 1—a remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ( $3,200 \div 8 = 400$ )

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

Step 1 continued...

$$\begin{array}{r} \text{h t o} \\ 061 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply  $1 \times 4 = 4$ , write that four under the 7, and subtract. This finds us the remainder of 3.

Check:  $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0402 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply  $2 \times 4 = 8$ , write that eight under the 9, and subtract. This finds us the remainder of 1.


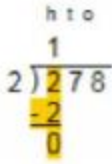
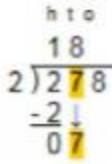
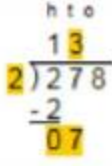
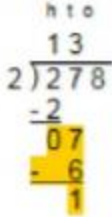
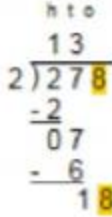
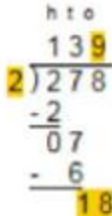
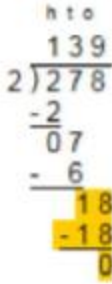
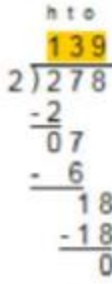
Check:  $4 \times 402 + 1 = 1,609$

Step 2—a remainder in the tens

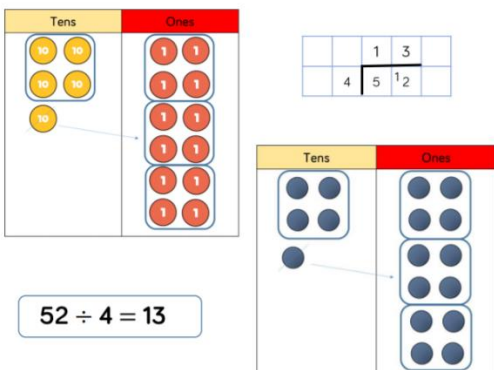
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \\ 2 \overline{) 58} \end{array}$ <p>Two goes into 5 two times, or 5 tens <math>\div 2 = 2</math> whole tens -- but there is a remainder!</p>	$\begin{array}{r} \text{t o} \\ 2 \\ 2 \overline{) 58} \\ \underline{-4} \\ 1 \end{array}$ <p>To find it, multiply <math>2 \times 2 = 4</math>, write that 4 under the five, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \downarrow \\ 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.</p>

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>Multiply <math>9 \times 2 = 18</math>, write that 18 under the 18, and subtract.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>The division is over since there are no more digits in the dividend. The quotient is 29.</p>

Step 2 – A remainder in any of the place values.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
 <p>Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.</p>	 <p>Multiply <math>1 \times 2 = 2</math>, write that 2 under the two, and subtract to find the remainder of zero.</p>	 <p>Next, drop down the 7 of the tens next to the zero.</p>
Divide.	Multiply & subtract.	Drop down the next digit.
 <p>Divide 2 into 7. Place 3 into the quotient.</p>	 <p>Multiply <math>3 \times 2 = 6</math>, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	 <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
 <p>Divide 2 into 18. Place 9 into the quotient.</p>	 <p>Multiply <math>9 \times 2 = 18</math>, write that 18 under the 18, and subtract to find the remainder of zero.</p>	 <p>There are no more digits to drop down. The quotient is 139.</p>

**Skill: Divide 2-digits by 1-digit (grouping)**      **Year: 4/5**



When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

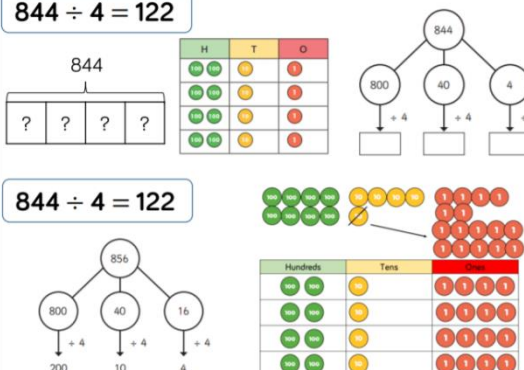
Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

Remainders can also be seen as they are left ungrouped.

$52 \div 4 = 13$

**Skill: Divide 3-digits by 1-digit (sharing)**      **Year: 4**

$844 \div 4 = 122$



Children can continue to use place value counters to share 3-digit numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows. This method can also help to highlight remainders. Flexible partitioning in a part-whole model supports this method.

$844 \div 4 = 122$

**Skill: Divide 3-digits by 1-digit (grouping)** **Year: 5**

Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

Place value counters or plain counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group them through a more pictorial method.

$856 \div 4 = 214$

**Skill: Divide 4-digits by 1-digit (grouping)** **Year: 5**

Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method.

Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

$8,532 \div 2 = 4,266$

**Skill: Divide multi digits by 2-digits (short division)** **Year: 6**

When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

$432 \div 12 = 36$

$7,335 \div 15 = 489$

15	30	45	60	75	90	105	120	135	150
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**Skill: Divide multi-digits by 2-digits (long division)** **Year: 6**

Children can also divide by 2-digit numbers using long division.

Children can write out multiples to support their calculations with larger remainders.

Children will also solve problems with remainders where the quotient can be rounded as appropriate.

$432 \div 12 = 36$

$7,335 \div 15 = 489$

**Skill: Divide multi digits by 2-digits (long division)** **Year: 6**

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question.

Children can also answer questions where the quotient needs to be rounded according to the context.

$372 \div 15 = 24 \text{ r}12$

$372 \div 15 = 24 \frac{4}{5}$